

Magnetized strange quark matter in a mass-density-dependent model

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We investigate the properties of strange quark matter in a strong magnetic field with quark confinement by the density dependence of quark mass considering the total baryon number conservation, charge neutrality and chemical equilibrium. The strength of the magnetic field considered in this article is $10^{16} \sim 10^{20}$ G. It is found that an additional term should appear in the pressure expression to maintain thermodynamic consistency. At fixed density, the energy density of magnetized strange quark matter varies with the magnetic field strength. There exists a minimum with increasing the field strength, depending on the density. It is about 6×10^{19} Gauss at two times the normal nuclear saturation density.

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I. INTRODUCTION

Strange quark matter (SQM) is a form of matter which contains a large quantity of deconfined quarks in β -equilibrium, with electric and color charge neutrality. The conjecture that SQM could be the true ground state of strong interactions was proposed by Bodmer-Wittern-Terazawa [1–3]. Farhi and Jaffe [4] studied SQM in the framework of the MIT bag model [5] for various values of the strange quark mass and the bag constant. SQM could be succeeded in the inner core of neutron stars where strange quarks would be produced through the weak processes with a dynamical chemical equilibrium among the constituents. It is possible that after a supernova explosion its core forms directly a strange quark star (SQS) [6, 7] self-bound by strong interactions while in an general scenario neutron stars are bound by gravitational force. An astrophysical object could also form a hybrid star that has a quark core and a crust of hadronic matter.

The stability of SQM is strongly affected in a strong magnetic field [8]. In a real astrophysical scenario, the strong magnetic field plays an important role. The typical strength on the surface of pulsars could be of the order $\sim 10^{12}$ G. Magnetars and neutron stars could be associated with sources with intense magnetic fields around $\sim 10^{13} - 10^{15}$ G or even higher [9, 10]. The origin of such ultrastrong magnetic fields could be explain in two ways. One is that the magnetohydrodynamic dynamo mechanism with large magnetic fields generated by rotating plasma of a protoneutron star [10]. The other is that during the star collapse with magnetic flux conservation, the relatively small magnetic fields were amplified [11]. In a recent research, it was found that noncentral high-energy heavy-ion collisions could generate intense magnetic fields as high as about 10^{19} G, corresponding to $eB_m \sim 6m_\pi^2$, where e is the fundamental electric charge and m_π is the pion mass.

Since the lattice approach still has difficulty in consistent treatment of the finite chemical potential and the application of perturbative quantum chromodynamics (QCD) to the strong-coupling domain is unbelievable, we have to use phenomenological models in most cases. In Ref. [8], Chakrabarty studied quark matter in a strong magnetic field with conventional MIT bag model, and found that SQM becomes more stable if the order of the strength of the magnetic field is greater than some critical value. In Refs. [12, 13], the authors confirmed that there is an anisotropy of pressures due to the strong magnetic field [14–16] and that the MIT bag model can be used to study magnetized SQM (MSQM)satisfactorily. With the Nambu-Jona-Lasinio (NJL) model the properties of mSQM were also discussed by many researchers [17–21]. The linear sigma model coupled to quarks and to the Polyakov loop was used to investigate the influence of strong magnetic field background on confining and chiral properties of QCD [22], where the impact of the vacuum correction from quarks on the QCD phase structure was discussed.

In nuclear physics it is well known that particle masses vary with environment. Such masses are called effective masses [24–27]. Effective masses and effective bag constants for quark matter had been broadly studied [28, 29]. In quasiparticle model, the particle mass is derived at the zero-momentum limit of the dispersion relations from an effective quark propagator by resumming one-loop self-energy diagrams in the hard-dense-loop approximation. This reveals the dependence of particle masses on chemical potentials. In a recent research [30], the authors extended the quasiparticle model to studying MSQM. They find a density- and magnetic-field-dependent bag function, which has a maximum at 2 – 3 times the saturation density when the QCD scale parameter is larger than 123 MeV.

In the present paper, we apply another quark model with confinement by the density dependence of quark masses. In this model the masses of quarks depend upon the baryon number density. This idea was initially introduced by Fowler, Raha, and Weiner and was used to study light quark matter [31]. Later Chakrabarty and coworkers applied the model to the case of SQM [32–34]. The main advantage of CDDM model is the inclusion of quark confinement without using the bag constant. Instead, it is achieved by the density dependence of the quark masses derived from in-medium chiral condensate [35–37]. The two most important concentrations in this model are the quark mass scaling [35, 37] and the thermodynamic treatment [36, 37]. In the beginning, the interaction part of the quark masses was assumed to be inversely proportional to the density [31, 32]. Researchers also suggested other mass scalings [38, 39]. In Ref. [35] and [37], a cubic root scaling was derived based on the in-medium chiral condensates and linear confinement at zero and finite temperature respectively. This scaling has been used to investigate many aspects of SQM [40–43]. In the present article, we use the CDDM model with cubic root mass scaling to study the properties of SQM in a strong magnetic field.

The article is organized as follows. In Sec. II, we derive the thermodynamic formulas when quark masses are density dependent. In Sec. III, we analyze the properties of mSQM and present our numerical results. The effects of the magnetic field on the system are also discussed. In Sec. IV, the mass-radius relation of magnetized quark stars is investigated. A short summary is presented in Sec. V.

II. THERMODYNAMICS TREATMENT

In the CDDM model, quark confinement is achieved by the density dependence of quark masses: with decreasing density, the mass of a quark becomes infinitely large so that the vacuum is unable to support it. Therefore, the proper form of the density dependence is very important. Originally, the quark masses are parameterized as

$$m_q = m_{q0} + \frac{B}{3n}, \quad (1)$$

where $q = u, d, s$ quarks, m_{s0} is the current mass of flavor q , B is the famous MIT bag constant, n is the baryon number density.

Based on the in-medium chiral condensates, a cubic root scaling was derived at zero temperature [35], and it has been recently extended to finite temperature [37]. We adopt this mass scaling in the present article. At zero temperature, the mass scaling is

$$m_q = m_{q0} + \frac{D}{n^{1/3}}, \quad (2)$$

where m_{q0} is the quark current mass, D is the confinement parameter, n is the total baryon number density, the exponent of density was derived based on the in-medium chiral condensates and linear confinement at zero temperature [35]. In the present model, the parameters are the electron mass m_e , the quark current masses m_{u0}, m_{d0}, m_{s0} , and the confinement parameter D . The electron does not participate in strong interaction, its mass is a constant, i.e. $m_e = 0.511$ MeV. Since the light-quark current masses are too small compared to the interaction part, we take $m_{u0} = m_{d0} = 0$. The strange quark current mass is 95 ± 25 MeV [44]. The parameter D has been discussed in Ref. [36, 37, 45]. We treat D as a free parameter here. The value of $D^{1/2}$ should be in the range (156, 270) MeV [37, 45].

The starting point of our manipulation for SQM in a strong magnetic field is the energy density. Let's start with the energy density expression as a free particle system

$$E = \sum_i E_i = \sum_i \frac{2g_i}{(2\pi)^3} \int \int \int \sqrt{p^2 + m_i^2} d^3\vec{p}, \quad (3)$$

where the sum is over u, d, s quarks and the electron, g_i is the degeneracy factor ($g_i = 3$ (color) for quarks and $g_i = 1$ for electrons). The degeneracy due to spin has been denoted by factor 2 in the numerator.

We assume the magnetic field to be directed along the z axis with constant field strength B_m . The single particle energy spectrum is given by [46]

$$\epsilon_{n,s}^{(i)} = \sqrt{p_z^2 + m_i^2 + e_i B_m (2n + s + 1)}, \quad (4)$$

where $n = 0, 1, 2, \dots$, are the principal quantum numbers for allowed Landau levels, e_i is the absolute value of the electronic charge (e.g., $e_u = 2/3$, $e_d = e_s = 1/3$, and $e_e = 1$), $s = \pm 1$ refers to spin-up or spin-down states, and p_z is the component of particle momentum along the direction of the external magnetic field. We now should replace the integration over $p_x - p_y$ plane in the momentum space by the rule [8]

$$\int \int dp_x dp_y \longrightarrow 2\pi e_i B_m \sum_{s=\pm 1} \sum_n, \quad (5)$$

Notice that the $(0, -1)$ state is a nondegenerate state while $(n_0, 1)$ state and $(n_0 + 1, -1)$ degenerate, we can simplify the notation by setting $2\nu = 2n + s + 1$, where $\nu = 0, 1, 2, \dots$. Then the single particle energy spectrum becomes

$$\epsilon_{n,s}^{(i)} = \sqrt{p_z^2 + m_i^2 + 2\nu e_i B_m} \quad (6)$$

and the substituting rule in Eq. (5) was switched to

$$2\pi e_i B_m \sum_{s=\pm 1} \sum_n \longrightarrow 2\pi e_i B_m \sum_{\nu} (2 - \delta_{0\nu}). \quad (7)$$

Accordingly, the energy density of mSQM is given by

$$E = \sum_i E_i = \sum_i \frac{g_i e_i B_m}{4\pi^2} \sum_{\nu=0} (2 - \delta_{0\nu}) \int \sqrt{p_z^2 + m_i^2 + 2\nu e_i B_m} dp_z, \quad (8)$$

Explicitly carrying out the integration

$$E_i = \frac{g_i e_i B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \int_0^{p_{i,\nu}} \sqrt{p_z^2 + M_{i,\nu}^2} dp_z \quad (9)$$

$$= \frac{g_i e_i B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \left[\epsilon_i \sqrt{\epsilon_i^2 - M_{i,\nu}^2} + M_{i,\nu}^2 \ln \left(\frac{\sqrt{\epsilon_i^2 - M_{i,\nu}^2} + \epsilon_i}{M_{i,\nu}} \right) \right], \quad (10)$$

where ϵ_i is the Fermi energy for particle type i , $M_{i,\nu} = \sqrt{m_i^2 + 2\nu e_i B_m}$ is the quark effective mass in the presence of a magnetic field, $p_{i,\nu}$ is the maximum value of p_z for the energy level ν , satisfying the relation $\epsilon_i = \sqrt{p_{i,\nu}^2 + M_{i,\nu}^2}$. From the positive value requirement on the Fermi momentum we can determine the upper bound ν_{\max} of the summation index ν

$$\nu \leq \nu_{\max} \equiv \text{int} \left[\frac{\epsilon_i^2 - m_i^2}{2e_i B_m} \right], \quad (11)$$

where $\text{int}[A]$ means taking the integer part of A .

For a Fermi gas system, the number density is given by

$$n_i = \frac{g_i}{(2\pi)^3} \int d^3 \vec{p}. \quad (12)$$

In the presence of a magnetic field, according to the rule in Eq. (7), the expression of the number density can be apparently given as

$$n_i = \frac{g_i e_i B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) p_{i,\nu} \quad (13)$$

$$= \frac{g_i e_i B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \sqrt{\epsilon_i^2 - M_{i,\nu}^2}. \quad (14)$$

In order to obtain the equation of states and check the thermodynamic consistency, we use the thermodynamic relation between pressure and the chemical potentials μ_i :

$$P = -E + \sum_i \mu_i n_i, \quad (15)$$

which is valid for arbitrary infinitely large system. The chemical potentials are connected to the energy density by

$$dE = \sum_i \mu_i dn_i, \quad (16)$$

where

$$dE = \sum_i dE_i = \sum_i \left(\frac{\partial E_i}{\partial \epsilon_i} d\epsilon_i + \frac{\partial E_i}{\partial m_i} dm_i \right), \quad (17)$$

$$dn_i = \frac{\partial n_i}{\partial \epsilon_i} d\epsilon_i + \frac{\partial n_i}{\partial m_i} dm_i, \quad (18)$$

$$dm_i = \sum_j \frac{\partial m_i}{\partial n_j} dn_j. \quad (19)$$

Using these relations, we finally have

$$\mu_i = \epsilon_i + \mu_I, \quad (20)$$

with

$$\mu_I = -\frac{m_I}{9n} \sum_j \frac{g_j e_j B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) m_j \ln \left(\frac{\epsilon_j + \sqrt{\epsilon_j^2 - M_{j,\nu}^2}}{M_{j,\nu}} \right), \quad (21)$$

where $m_I = \frac{D}{n^{1/3}}$ is the interactive part in the mass scaling. We should notice that because all particle masses do not depend on the density of electrons, i.e. $\frac{\partial m_j}{\partial n_e} = 0$, the summation is just over u, d, s and the term as a whole is independent of quark flavors. Substituting Eq. (10), Eq. (14), Eq. (20), Eq. (21) into Eq. (15) gives

$$P = -\Omega_0 + \delta P, \quad (22)$$

with

$$\Omega_0 = - \sum_i \frac{g_i e_i B_m}{4\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) \left\{ \epsilon_i (\epsilon_i^2 - M_{i,\nu}^2)^{1/2} - M_{i,\nu}^2 \ln \left[\frac{\epsilon_i + \sqrt{\epsilon_i^2 - M_{i,\nu}^2}}{M_{i,\nu}} \right] \right\}, \quad (23)$$

and

$$\delta P = \sum_i n_i \mu_I = 3\mu_I n = -\frac{m_I}{3} \sum_j \frac{g_j e_j B_m}{2\pi^2} \sum_{\nu=0}^{\nu_{\max}} (2 - \delta_{\nu 0}) m_j \ln \left(\frac{\epsilon_j + \sqrt{\epsilon_j^2 - M_{j,\nu}^2}}{M_{j,\nu}} \right), \quad (24)$$

where Ω_0 is the thermodynamical potential density for a free-particle system in the presence of a magnetic field.

Comparing Eq. (22) to constant-mass case where

$$P = -\Omega_0, \quad (25)$$

we notice that the term δP , introduced by the density dependence of quark masses, guarantees the Hugenholtz-Van Hove theorem.

III. PROPERTIES OF MSQM

In this section, we carry out a numerical study to investigate the relevant thermodynamical quantities for mSQM, taking into account β equilibrium and charge neutrality. As usually done, we consider mSQM as a mixture of u, d, s quarks and electrons. The weak equilibrium condition gives

$$\mu_u + \mu_e = \mu_d, \quad (26)$$

$$\mu_d = \mu_s. \quad (27)$$

Considering Eq. (20), we have

$$\epsilon_u + \epsilon_e = \epsilon_d, \quad (28)$$

and

$$\epsilon_d = \epsilon_s. \quad (29)$$

The charge neutrality condition gives

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0, \quad (30)$$

and the baryon number density is given by

$$n = \frac{1}{3}(n_u + n_d + n_s). \quad (31)$$

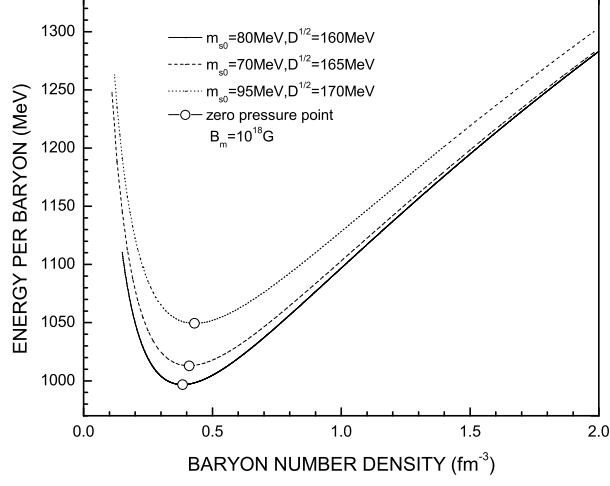


Figure 1. Energy per baryon of strange quark matter in the present model. The energy minimum is located exactly at the same point of the zero pressure.

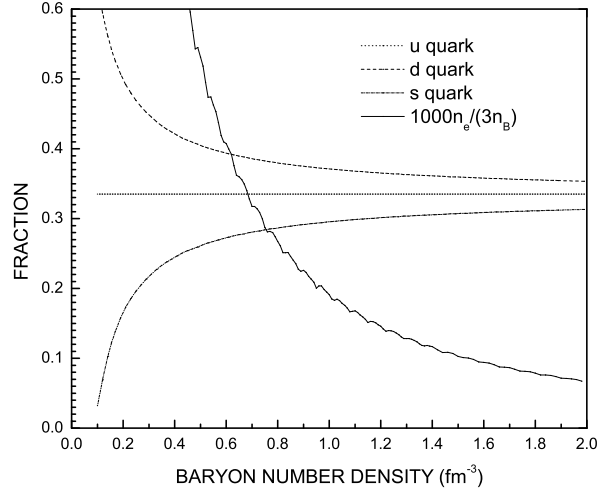


Figure 2. Quark fraction vs. baryon number density for $D^{1/2} = 160$ MeV, $m_{s0} = 80$ MeV and $B_m = 10^{18}$ G.

Eqs. (28), (29), (30) and (31), together with Eq. (14), form the full set of self-consistency equations for finding the Fermi energy for quarks and electrons. We solve these equations numerically to obtain the Fermi energies of each particle type at a given density n and for different values of B_m , and then evaluate the thermodynamical quantities of the MSQM system.

In Fig. 1, the energy per baryon of MSQM is shown as functions of different parameter sets. For each parameter set the pressure is zero at the minimum the energy. In fact from Eq. (15) and Eq. (16) one can obtain $P = n^2 \frac{d(E/n)}{dn}$. This relation guarantees the thermodynamic consistency. In the present paper, we assume the magnetic field to be directed along the z axis and is a constant. If one allows the magnetic field to vary with the density, one should add to the chemical potential another new term. The discussion about the details of the magnetic field variation with the baryon number density is beyond this paper.

In Fig. 2, we plot the quarks fractions, i.e., $n_u/(3n)$, $n_d/(3n)$, $n_s/(3n)$, and the 10^3 times the electron fraction, $1000n_e/(3n)$, versus the baryon number density for $D^{1/2} = 160$ MeV and $m_{s0} = 80$ MeV. The magnetic field strength

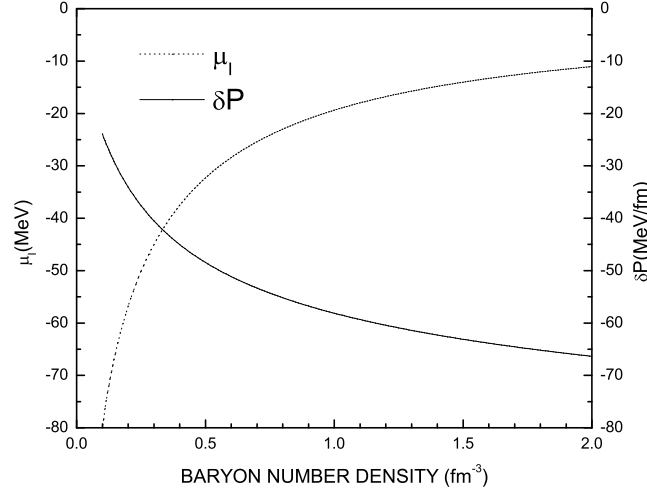


Figure 3. The terms introduced by the density dependence of quark masses for pressure and chemical potential versus baryon number density. $D^{1/2} = 160$ MeV, $m_{s0} = 80$ MeV and $B_m = 10^{18}$ G.

is $B_m = 10^{18}$ G. The fraction of up quarks is always about one third. The fraction of down quarks decreases with increasing density while the fraction of strange quarks increases with increasing density. Both fractions approach one third when the baryon number density is large enough. The figure indicates that the fraction of electrons is very small and it decreases with increasing baryon number density. The ladder-like shape is introduced by the quantized laudau levels. We can expect that when the baryon number density increases to large enough value even the current mass of strange quark matter can be ignored and all three kinds of quarks can be treated equivalently. This is easy to understand. For fermions at zero temperature, the states below Fermi energy are all occupied. Adding particles into the system is just enlarge the Fermi energy, which is equivalent to magnify the Fermi momentum. At the same time the effective mass decreases with density. When the Fermi momentum is sufficiently large, the masses of most particles are ignorable compared to their momenta and we can treat them as if their effective masses are zero.

To explicitly demonstrate the properties of μ_I and δP , we plot them in Fig. 3. The additional term of the chemical potential tends to reduce the total value, but the effects gradually weaken when the baryon number density increases. For δP , it is a negative number and the curve shows a increasing tendency of the absolute value with baryon number density. From this set of curves we find that for respective type of particles the interactive part decreases with increasing the particle number density. When the density reach a sufficiently large value, the interactive effect can be ignored and the chemical potentials get close to the Fermi energy, which means that we can treat each type of fermions as free particles. But when dealing with the overall effect, the pressure, the interactive effect introduced by the dependence of masses on particle density, always exists and the effect of additional term reduces the pressure of the system.

In Fig. 4, we show how the energy density of mSQM varies with the magnetic field strength at given densities $n = 2n_0$ (dotted curve) and $n = 3n_0$ (solid curve). For small B_m , it is very obvious that the energy density of mSQM is nearly constant. When the magnetic field strength exceeds a critical value, which is about 3×10^{18} G for $n = 2n_0$, the energy density begins to decrease, until a minimum is reached. The minimum depends on the density. It is 6×10^{19} G for $n = 2n_0$. If one would hope to include the pure magnetic field term $B_m^2/2$, as shown in Fig. 5, the minimum disappears and the total energy density increases monotonously. This is because the magnetic field itself becomes dominant at extremely strong strength.

In Fig. 6 we show the pressure as a function of the strength of magnetic field for $n = 3n_0$ (solid line) and $n = 2n_0$ (dash line). At the beginning the pressure stays nearly constant and starts to decrease apperantly at 2×10^{18} G. Since we using the CDDM model to investigate the properties of mSQM, we have to assume the pressure to be isotropic because the Tolman-Oppenheimer-Volkoff (TOV) equations as usual method for finding mass-radius relationship only fits spherically symmetrical and static compact star. In Ref. [47], the authors applied anisotropic pressure to strange quark matter system. The properties of the pressure in present article is in accordance with the longitudinal pressure in Ref. [47].

Fig. 7 and Fig. 8 show the chemical potentials as functions of the magnetic field for $n = 2n_0$ and $n = 3n_0$

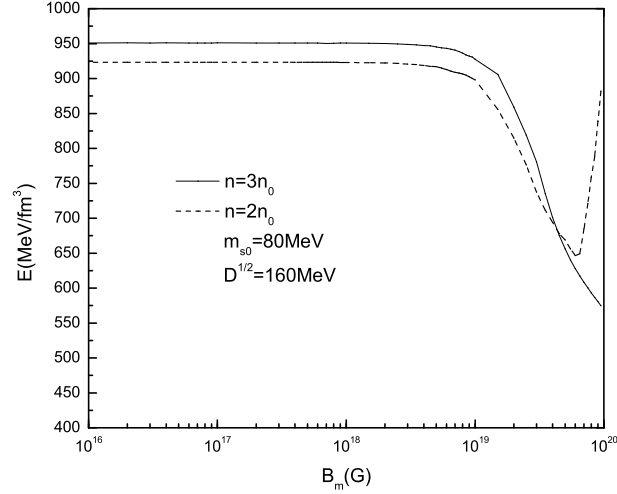


Figure 4. Energy density of mSQM as a function of the magnetic field strength. The parameter pair $(m_{s0}, D^{1/2})$ in MeV is (80, 160). The solid line is for $n = 3n_0$ and the dash line is for $n = 2n_0$ ($n_0 = 0.16 \text{ fm}^{-3}$).

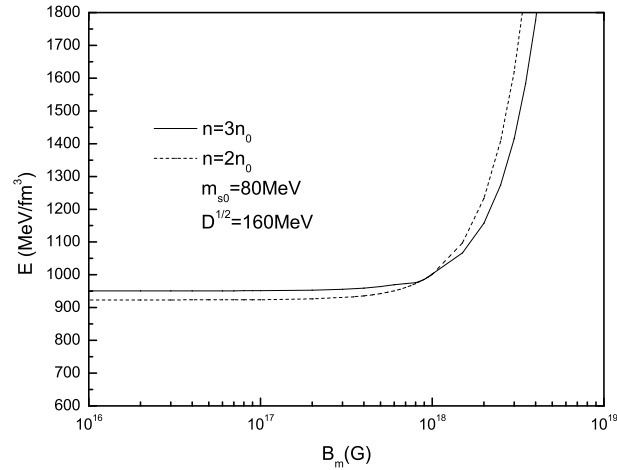


Figure 5. Energy density plus that of the magnetic field as a function of magnetic field strength. The parameter pair $(m_{s0}, D^{1/2})$ in MeV is (80, 160). The solid line is for $n = 3n_0$ and the dash line is for $n = 2n_0$ ($n_0 = 0.16 \text{ fm}^{-3}$).

respectively. The chemical potentials keep approximately constant with increasing magnetic field for $B_m \leq 10^{18} \text{ G}$. The apparent Landau oscillation of chemical potential appears between $3 \times 10^{18} \text{ G} \sim 2 \times 10^{19} \text{ G}$. At $B_m \geq 3 \times 10^{19} \text{ G}$ quark chemical potentials decrease with magnetic field. We notice that in both figures the chemical potential of electron dramatically climbs from its initial relatively small value to its summit (about 80 MeV) in the interval $2 \times 10^{19} \text{ G} \leq B_m \leq 3 \times 10^{19} \text{ G}$, then decreases to a relatively high and even stage (about 60 MeV). We have not encountered negative chemical potential for electron which contrasts with the situation in Ref. [47].

IV. MASS-RADIUS RELATION OF MAGNETIZED STRANGE QUARK STARS

It has been speculated that the currently named neutron stars may in fact be strange quark stars [48], a family of compact stars consisting completely of deconfined u, d, s quarks. The structure of strange quark stars has attracted

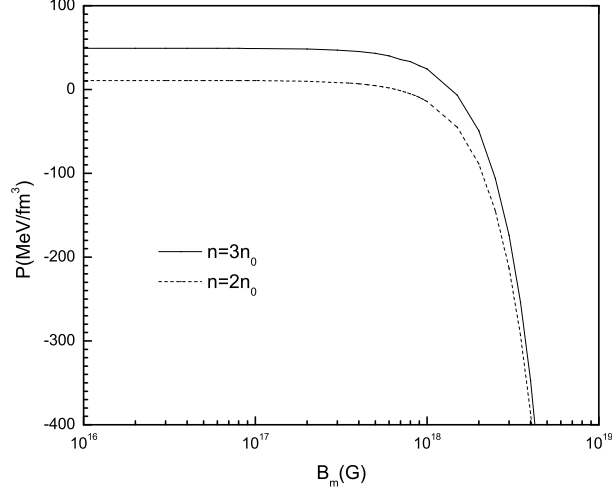


Figure 6. Pressure in mSQM at zero temperature as functions of the magnetic field strength for $m_{s0} = 80$ MeV and $D^{1/2} = 160$ MeV.

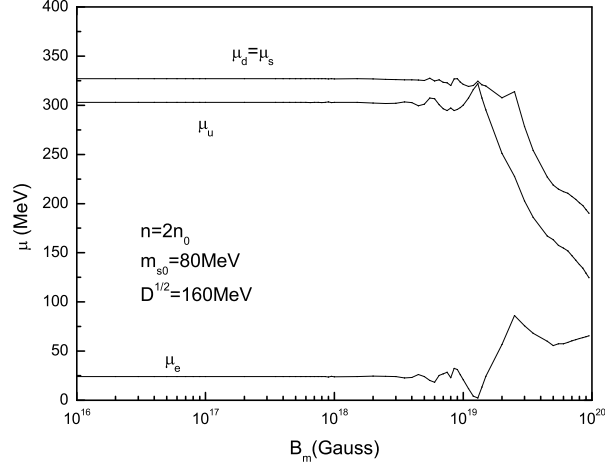


Figure 7. Chemical potentials as function of the magnetic field strength at zero temperature for $n = 2n_0$.

plenty of researchers. We investigate the mass-radius relation of strange quark stars in ordinary phase in the framework of the new EOS we obtained in the preceding section.

We follow the method that numerically solving Toman-Oppenheimer-Volkoff (TOV) equations when fixing a central pressure P_c , and obtain a mass-radius relation by continuously varying the central pressure. For a concrete description of the solving process, one may refer to Ref. [36]. Since the method only fits spherically symmetrical and static compact star, we have assume the pressure to be isotropic in the entire article. From Sec. III we find that this assumption functions well at $B_m \lesssim 3 \times 10^{18}$ G. We alter parameter D and magnetic field strength B_m to see how can the model and the magnetic field affect the structure of strange quark stars. The results have been showed in Fig. 9. The full dots represent the maximum mass for each line.

We can see that enhanced B_m or parameter D could reduce the maximum mass. According to Fig. 5 and Fig. 6, enhancing the magnetic field strength will increases energy density and decreases pressure which means softer EOS. Therefore the present EoS is unstable to produce the maximum star mass as large as two times the solar mass ($2M_\odot$) [49, 50]. This is due to the fact that the quark mass scaling used in the present calculations includes only

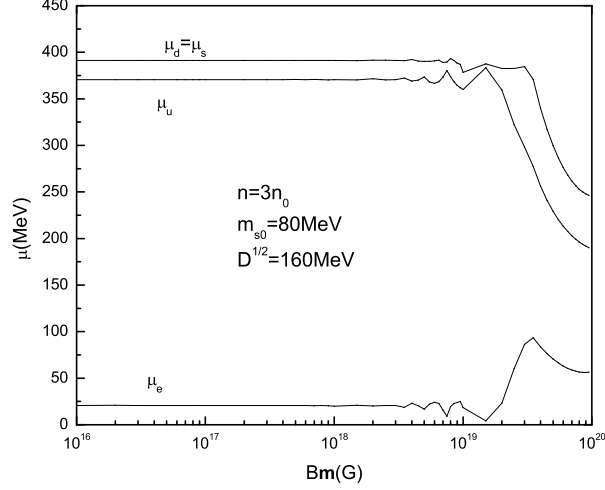


Figure 8. Chemical potentials as function of the magnetic field strength at zero temperature for $n = 3n_0$.

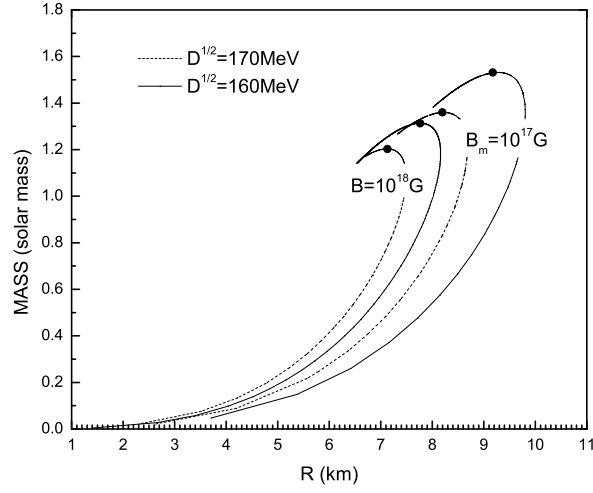


Figure 9. The mass-radius relation of SQS at different magnetic field B_m with $D^{1/2} = 160$ MeV (solid lines) and $D^{1/2} = 170$ MeV (dash lines). The maximum masses on are marked by full dots.

the confinement interaction effects whose contribution to the pressure is negative, while the perturbative interactions become important at high density. Therefore, it is meaningful to deduce a quark mass scaling considering both the confinement and perturbative effects [50], which is an urgent forthcoming task in the near future.

V. SUMMARY

We have extended the CDDM model with a cubic root mass scaling to study the properties of strange quark matter in the presence of a strong magnetic field. Our thermodynamic treatment automatically guarantees the self-consistency. It is found that at high density quarks of different kinds can be treated equivalently for dynamic properties and individual particle acts like free particle while the overall effect introduced by the density dependence of quark masses always exist. The magnetic field will reduce the energy density of pure magnetized strange quark matter at

certain range of the field strength. At $B_m \leq 10^{18}$ G, magnetic field affects the properties of the system slightly. At $10^{18} \text{ G} \leq B_m \leq 10^{19} \text{ G}$, Landau oscillation appears in chemical potential and the pressure dramatically decreases. At $B_m \approx 4 \times 10^{18} \text{ G}$ the energy density of pure magnetic field becomes comparable with, and finally much large than that of the pure magnetized SQM. With the obtained EoS, we study the mass-radius relation of quark stars. It is found that one can not produce a pure quark star with mass as large as $2M_\odot$ considering only the confinement interaction in the quark mass scaling. A quark mass scaling with both the confinement and perturbative interactions are needed.

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